

MTH241 Fall 2024: Exam 01

Instructor: Dr. Ling Liang

Date: September 25, 2024

Time: 3:00pm - 3:50pm

Name:

UID:

Closed book, no calculator, show your work clearly.

1. (15pt) Mark True or False for each of the following statements. (**Grading:** 3pt each)
- (a) Let P, Q, R be any three points in the space, then $\|\overrightarrow{PQ}\| \leq \|\overrightarrow{PR}\| + \|\overrightarrow{RQ}\|$.
Your answer: True False
- (b) Let \vec{a} be a vector and c be any number, then $\|c\vec{a}\| = c\|\vec{a}\|$.
Your answer: True False
- (c) Let \vec{a} and \vec{b} be nonzero vectors and θ be the angle between them. Then, $\vec{a} \bullet \vec{b} = \|\vec{a}\|\|\vec{b}\|\sin(\theta)$.
Your answer: True False
- (d) Any three points in the spaces that are not on the same line uniquely determine a plane.
Your answer: True False
- (e) Any smooth curve is also piecewise smooth.
Your answer: True False
2. (30pt) Let $\vec{a} = -\vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = -2\vec{i} + \vec{j} - 3\vec{k}$.
- (a) Compute $\vec{a} \bullet \vec{b}$, $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$. (**Grading:** 4pt for working, 3pt for correct dot product, 4pt for each of the correct cross products)

(b) Compute the projection of \vec{b} onto \vec{a} . (**Grading:** 4pt for working, 2pt for correct norm, 4pt for correct projection)

(c) Find an equation of the plane that contains a point $P_0 = (-1, -2, -1)$ and is parallel to both \vec{a} and \vec{b} . (**Grading:** 5pt for working, 5pt for correct equation)

3. (30pt) Consider the following equations for the line ℓ :

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}.$$

- (a) Find a point on the line and a vector that is parallel to the line. (**Grading:** 3pt for working, 6pt for correct point, 6pt for correct vector)

- (b) Let $P_1 = (1, 1, 1)$, find the distance D from P_1 to the line ℓ . (**Grading:** 5pt for working, 5pt for correct formula, 5pt for correct distance)

4. (25pt) Let $\vec{r}(t) = \cos(t)\vec{i} + \sin(t)\vec{j} + \frac{2}{3}t^{3/2}\vec{k}$ for $t \in [0, 1]$ be the parameterization of a curve C .

(a) Determine if C is smooth on $[0, 1]$. (**Grading:** 4pt for working, 5pt for correct criterion of smoothness, 3pt for correct answer)

(b) Compute the length L of the curve C defined on $[0, 1]$. (**Grading:** 5pt for working, 5pt for correct formula, 3pt for correct distance.)

Extra page: